جامعة بيرزيت

BIRZEIT UNIVERSITY

Electrical and Computer Engineering

ENEE2302 - Signals and Systems

Midterm Exam – Spring 2015

Instructor: Dr. Abualsoud Hanani

Date: 30/3/2015

Name: Key

Id:

Question	Max Mark	Mark	ABET outcome
1	**20**		a
2	**20**		k
3	**20**		a
Bonus	5		k
Total	**60**		

$$\exp(j\theta) = \cos\theta + j\sin\theta$$
$$\exp(j\theta) + \exp(-j\theta) = 2\cos\theta$$
$$\exp(j\theta) - \exp(-j\theta) = 2j\sin\theta$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$
$$2\sin A\sin B = -\cos(A+B) + \cos(A-B)$$
$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Fourier Transform pairs:

$$\delta(t) \leftrightarrow 1$$

$$u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$$

$$\mathrm{rect}\left(\frac{t}{\tau}\right) \leftrightarrow \tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$

$$\frac{B}{2\pi} sinc\left(\frac{Bt}{2}\right) \leftrightarrow rect\left(\frac{\omega}{B}\right)$$

$$\cos(\omega_0 t) \leftrightarrow \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

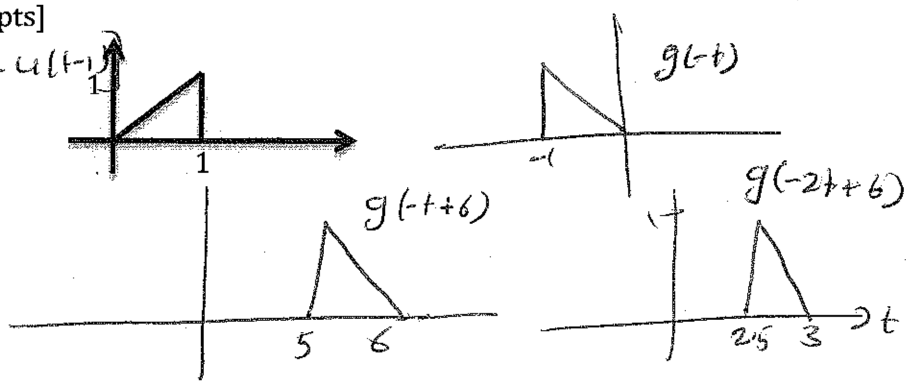
$$\sin(\omega_0 t) \leftrightarrow \frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]]$$

Question 1 [20pts]:

a) Given a signal g(t) shown in the following figure, express it in terms of singularity signals? And plot g(-2t+6)?[5pts]

$g(t) = t\,[u(t) - u(t-1)]$

$g(-(2t-6))$



b) Evaluate the following integral: $\int_{-\infty}^{\infty}(3t^2 + e^{-5t})\ddot{\delta}(t-2)dt$. [5pts]

$$\left.\frac{d^2(3t^2 + e^{-5t})}{dt^2}\right|_{t=2} = \left.(6 + 25e^{-5t})\right|_{t=2} = 6 + 25e^{-10}$$

c) If $x(t) = 4\sin\left(\frac{\pi}{3}t\right) + \cos\left(\frac{\pi}{4}t - \frac{\pi}{6}\right)$. Is x(t) periodic? If so, find its fundamental period? What are its harmonics? [5pts]

$T_1 = \frac{2\pi}{\pi/3} = 6$, $T_2 = \frac{2\pi}{\pi/4} = 8$ $\Big\}$ $\frac{T_1}{T_2} = \frac{6}{8} = \frac{3}{4}$ rational $\Rightarrow$ x(t) is periodic

$\frac{\omega_1}{\omega_0} = \frac{\pi/3}{\pi/12} = 4 \rightarrow$ fourth harmonic

$\omega_0 = \frac{\pi}{12} \Leftarrow T_0 = 4T_1 = 4 \times 6 = 24$ sec

$\frac{\omega_2}{\omega_0} = 3 \Rightarrow$ Third harmonic

d) Is x(t) in part c) above energy or power signal? If it is energy find its energy, if it is power find its average power? [5pts]

Power signal [sinusoidal].

avg. Power $= \frac{A^2}{2}$ for each sinusoid.

so, $P = \frac{4^2}{2} + \frac{1^2}{2} = 8.5$ Watt

Question 2 [20 pts]:

a) Find impulse response of a system described by the following differential equation: [10pts]

$$\frac{d^3y(t)}{dt^3}+\frac{d^2y(t)}{dt^2}-2\frac{dy(t)}{dt}=\frac{dx(t)}{dt}+x(t)$$

$\lambda^3+\lambda^2-2\lambda=0$

$\lambda(\lambda+2)(\lambda-1)=0 \Rightarrow \lambda_1=0,\ \lambda_2=1,\ \lambda_3=-2$

$y_0(t)=c_1+c_2e^{t}+c_3e^{-2t}$

$y_0(0)=0;\ y_0'(0)=0,\ y_0''(0)=1$

$\boxed{c_1+c_2+c_3=0}$ — ①

$y_0'(t)=c_2e^{t}-2c_3e^{-2t}$

$y_0'(0)=\boxed{c_2-2c_3=0}$ — ②

$y_0''(t)=c_2e^{t}+4c_3e^{-2t}$

$y_0''(0)=\boxed{c_2+4c_3=1}$ — ③

Sub. ② from ③

$6c_3=1 \Rightarrow \boxed{c_3=\frac{1}{6}}$

$c_2=2c_3=\frac{1}{3}$

$c_1=-\left(\frac{1}{6}+\frac{1}{3}\right)=-\frac{1}{2}$

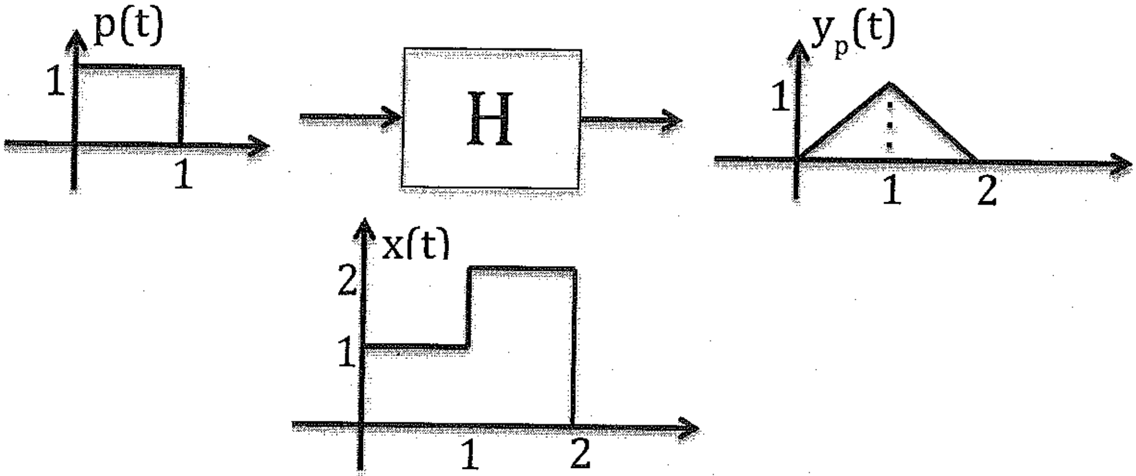
$y_0(t)=-\frac{1}{2}+\frac{1}{3}e^{t}+\frac{1}{6}e^{-2t}$

$h(t)=(P(D)y_0(t))u(t)=(D+1)\left(-\frac{1}{2}+\frac{1}{3}e^{t}+\frac{1}{6}e^{-2t}\right)u(t)$

$=\left(\frac{1}{3}e^{t}-\frac{1}{3}e^{-2t}-\frac{1}{2}+\frac{1}{3}e^{t}+\frac{1}{6}e^{-2t}\right)u(t)$

$=\left(\frac{2}{3}e^{t}-\frac{1}{6}e^{-2t}-\frac{1}{2}\right)u(t)$

b) Let H be a continuous-time linear and time-invariant (LTI) system, such that the system's response to a pulse input $p(t)=u(t)-u(t-1)$ is $H\{p(t)\}=y_p(t)$. Both signals are depicted in the following figures.



Given only the information above, we want to calculate the system's response to the input signal x(t) depicted in the figure above. Let's break down this problem into two parts:

i) Note that x(t) can be written as a sum of scaled and time-shifted of signal p(t). In particular,

$$x(t)=ap(t)+bp(t-t_0)$$

find adequate values for a, b and t_0? [3pts]

From figures above.

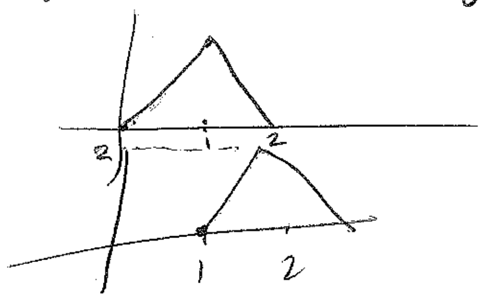
$a=1$

$b=2$

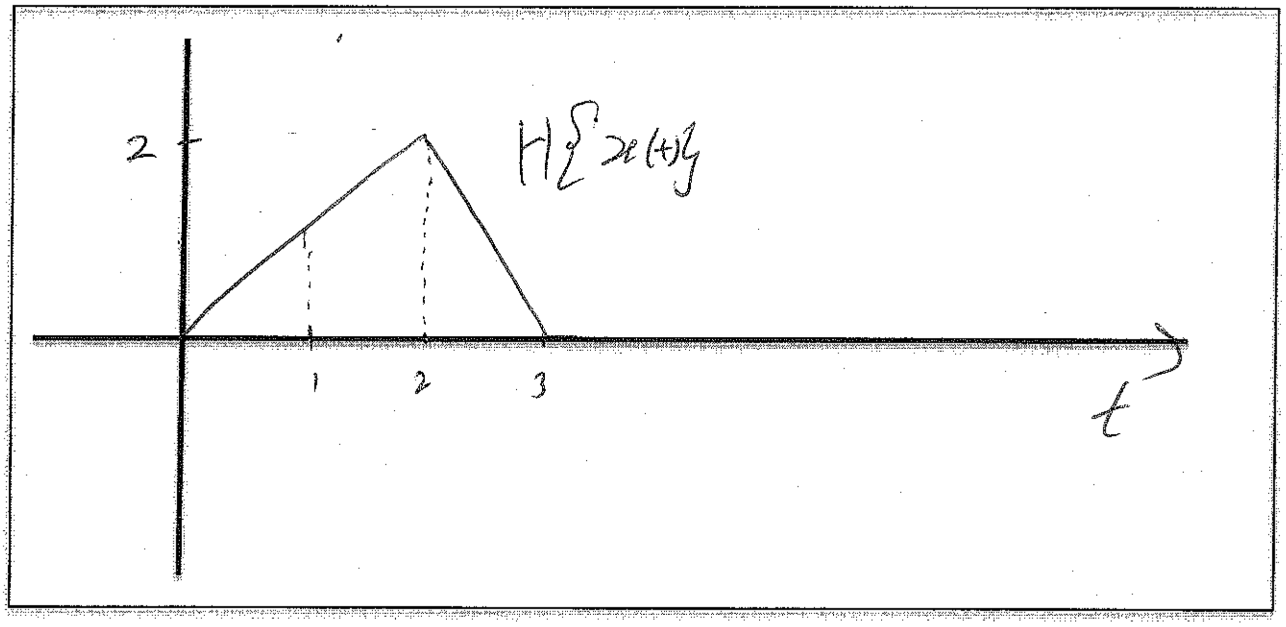
$t_0=1$

Since H is LTI $\Rightarrow$

$$H\{x(t)\} = H\{p(t)+2p(t-1)\} = H\{p(t)\} + 2\{p(t-1)\}$$



ii) Use what you know about the system and the result of part (a) to plot the system's response (output) to input signal x(t). [7pts]



Question 3 [20pts]:

a) For a time domain signal x(t) and its Fourier transform X(ω), drive and state

i) The time-scaling property. [2pts]

$$x(at) \xleftrightarrow{F} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

$F\{x(at)\} = \int_{-\infty}^{\infty} x(at) e^{-j\omega t} dt$, let $r = at \Rightarrow dt = \frac{1}{a} dr$

$$= \int x(r) e^{-j\frac{\omega r}{a}} \frac{1}{a} dr = \frac{1}{a} \int_{-\infty}^{\infty} x(r) e^{-j\frac{\omega}{a} r} dr$$

$$= \frac{1}{a} X\left(\frac{\omega}{a}\right)$$

ii) Shift property, where the shift is in the frequency domain [2pts]

$$e^{j\omega_0 t} x(t) \xleftrightarrow{F} X(\omega - \omega_0)$$

$$F\{e^{j\omega_0 t} x(t)\} = \int_{-\infty}^{\infty} e^{j\omega_0 t} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t) e^{-j(\omega-\omega_0)t} dt$$

$$= X(\omega - \omega_0)$$

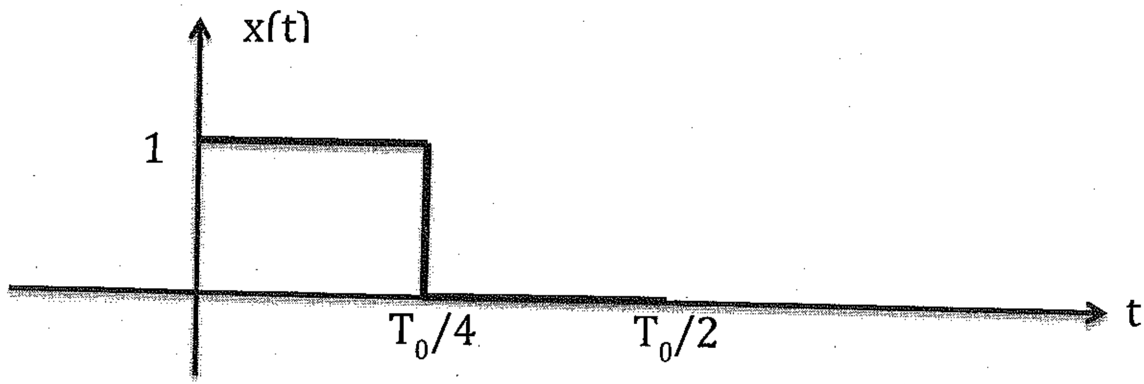
iii) Using the shift theorem, derive the modulation theorem, which gives the Fourier transform of $x(t)\cos(\omega_0 t)$ where ω_0 is a constant. [2pts]

$$F\{x(t)\cos(\omega_0 t)\} = F\{x(t)(\tfrac{1}{2}e^{j\omega_0 t} + \tfrac{1}{2}e^{-j\omega_0 t})\}$$

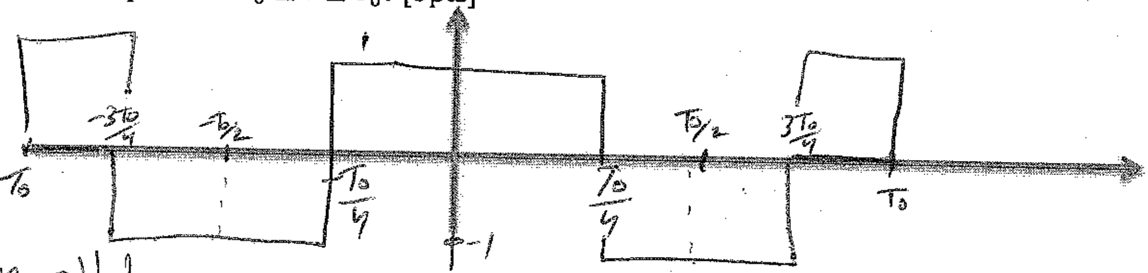
$$= \tfrac{1}{2}F\{x(t)e^{j\omega_0 t}\} + \tfrac{1}{2}F\{x(t)e^{-j\omega_0 t}\} \Rightarrow \text{from shift property in part (ii)}$$

$$= \tfrac{1}{2}X(\omega+\omega_0) + \tfrac{1}{2}X(\omega-\omega_0)$$

b) Suppose $x(t)$ is periodic signal with period T_0 and is specified in the interval $0 \le t \le \frac{T_0}{2}$ as shown in the figure below.



i) If Fourier series of this signal has only odd harmonics and $x(t)$ is even, sketch $x(t)$ in the period $-T_0 \le t \le T_0$? [3pts]



$*$ only odd harmonics means a_k or (a_n) must be nonzero for k odd only

$\Rightarrow a_0 = 0,\ a_2 = 0,\ a_4 = 0, \ldots$ etc.

$*$ also $x(t)$ must be even $\Rightarrow x(t - \frac{T_0}{2}) = -x(t)$

$$a_k = \frac{1}{T_0}\int_{-T_0/2}^{-T_0/4}(-1)\,e^{-jk\omega_0 t}\,dt + \frac{1}{T_0}\int_{-T_0/4}^{T_0/4}(1)\,e^{-jk\omega_0 t}\,dt + \frac{1}{T_0}\int_{T_0/4}^{T_0/2}(-1)\,e^{-jk\omega_0 t}\,dt$$

$T_0\omega_0 = 2\pi$

ii) Find an expression for its Fourier series coefficients, and plot them for n=-2, -1, 0, 1 and 2. [5pts]

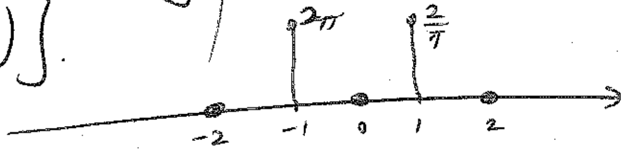
$$= \frac{1}{T_0}\cdot\frac{1}{jk\omega_0}\left\{\left[e^{-jk\omega_0 t}\right]_{-T_0/2}^{-T_0/4} - \left[e^{-jk\omega_0 t}\right]_{-T_0/4}^{T_0/4} + \left[e^{-jk\omega_0 t}\right]_{T_0/4}^{T_0/2}\right\}$$

$$= \frac{1}{jk2\pi}\left[e^{jk\frac{\pi}{2}} - e^{jk\pi} - e^{-jk\frac{\pi}{2}} + e^{jk\frac{\pi}{2}} + e^{-jk\pi} - e^{-jk\frac{\pi}{2}}\right]$$

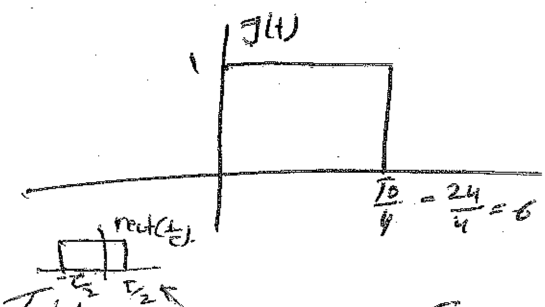
$$= \frac{1}{jk2\pi}\left[2e^{jk\frac{\pi}{2}} - 2e^{-jk\frac{\pi}{2}} - \left(e^{jk\pi} - e^{-jk\pi}\right)\right]$$

$$= \frac{2}{k\pi}\sin\left(\frac{k\pi}{2}\right) - \frac{1}{k\pi}\sin(k\pi)$$

$$a_1 = \frac{2}{\pi}\sin\left(\frac{\pi}{2}\right) - \frac{1}{\pi}\sin\pi = \frac{2}{\pi} = a_{-1}$$



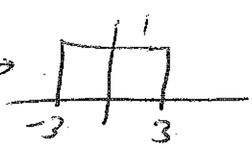
iii) If g(t) is a non-periodic signal defined as the positive half of the first period of x(t), find its Fourier Transform G(ω), if T_0=24 sec. [6pts]



From Table.

$$\mathrm{rect}\left(\frac{t}{\tau}\right) \xleftrightarrow{\mathcal{F}} \tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$

$x(t) \longrightarrow$



$$\xleftrightarrow{\mathcal{F}} 6\,\mathrm{sinc}\left(\frac{6\omega}{2}\right)$$

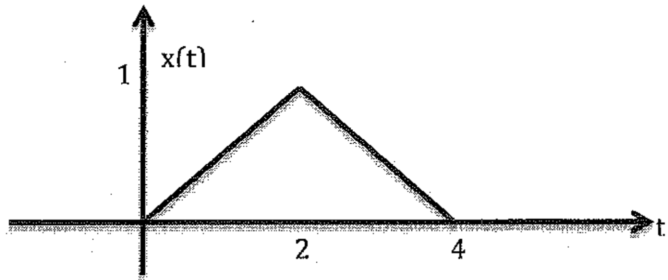
$x(t-3)$

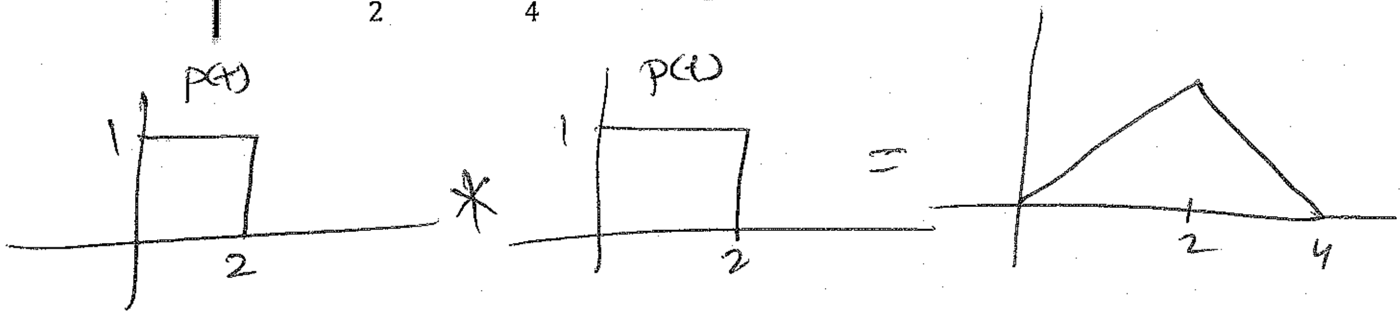
$$\xleftrightarrow{\mathcal{F}} 6\,e^{-j3\omega}\,\mathrm{sinc}(3\omega).$$

since $g(t)$ is shifted $\mathrm{rect}\left(\frac{t}{6}\right)$ by 3 $\Rightarrow$

$$\mathcal{F}\{g(t)\} = 6\,e^{-j3\omega}\,\mathrm{sinc}(3\omega).$$

Bonus [5pts]
Find Fourier Transfer of the following signal, x(t), using Fourier transform properties. Finding it using formal definition of Fourier transform (integral) or using table will not be accepted.





So, $x(t) = p(t) * p(t)$

$$\mathcal{F}\{x(t)\} = \mathcal{F}\{p(t) * p(t)\}.$$

$$\mathcal{F}\{p(t)\} = 2e^{-jw}\,\mathrm{sinc}(w). \;\left[\text{shift } \mathrm{rect}\left(\tfrac{t}{2}\right)\right].$$

$\Rightarrow$ Convolution in time-domain $\Rightarrow$ Multiplication in freq. domain

So, $\mathcal{F}\{p(t) * p(t)\} = \mathcal{F}\{p(t)\}\cdot\mathcal{F}\{p(t)\}$.

$$= \left(2e^{-jw}\,\mathrm{sinc}(w)\right)^2 = 4e^{-j2w}\,\mathrm{sinc}^2(w).$$

So, $\mathcal{F}\{x(t)\} = 4e^{-j2w}\,\mathrm{sinc}^2(w)$.